Structure of Falling Film Heat and Mass Transfer on a Fluted Tube

A. T. Conlisk
Dept. of Mechanical Engineering, Ohio State University, Columbus, OH 43210

The falling liquid film has become a popular means of transferring heat and mass from a vapor to a binary liquid, especially in gas-fired heat pump systems. Ideally, the required amount of heat and mass transfer can be accomplished by using a simple cylindrical tube; however, increasingly stringent size and weight requirements for the machine prohibit use of the simple cylindrical surface, and other more complex surfaces with higher absorption capacities have been sought. In this article, absorption of a single component and condensation of a binary mixture on an axially fluted tube is considered. The solution to the problem hinges on the energy equation, although the entire energy transfer process is mass-transfer-limited. Significant mass transfer is limited to a thin layer near the liquid-vapor interface. Solutions to the energy equation are obtained for both the conduction- and convection-dominated regimes. In the latter, significant heat transfer occurs within a thin layer near the liquid-vapor interface which contains the mass transfer layer; this "boundary layer" structure does not appear to have been recognized in previous work in this area. Using the present results, the capacity of a given tube may be predicted as a function of governing geometrical and physical parameters. The principal objective of this work is to develop the theoretical tools from which computations may be carried out during a design process. The theoretical results may be applied to mixtures typical of application in the absorption heat pump industry.

Introduction

The design of an absorption heat pump system usually begins with a thermodynamic analysis of a proposed cycle with a specific working fluid mixture; the outputs of this analysis are inlet and outlet mass fractions, and inlet and outlet temperatures associated with each component of the cycle along with the operating pressure. Due to its very nature, however, the thermodynamic analysis does not address whether any of the proposed states can be reached; thus, an additional step in the design process must involve fundamental experiments, and analysis and computation to determine whether a specific component design will meet system specifications. In this work, we consider the heat- and mass-transfer problems on a candidate enhanced surface for the absorber component in an absorption heat pump.

The falling liquid film has become a popular means of transferring heat and mass from a vapor to a binary liquid. Ideally, the required amount of heat and mass transfer can be accomplished by using a simple cylindrical tube (horizontal or vertical); however, increasingly stringent size and weight requirements for the heat pump, in general, prohibit use of the simple cylindrical surface, and other more complex surfaces have been sought which are believed to have a higher absorption capacity. The simplest "enhanced surface," in which significant improvement in heat transfer in pure fluids has been demonstrated, is the axially fluted tube, and the solution of the coupled heat and mass transfer on this tube is the objective of this article. The purpose of this work is to determine the structure of the flow, temperature, and mass fraction as a function of position within the liquid film for a wide range of operating parameters; the results may then be used to develop a design procedure whereby the amount of vapor absorbed or condensed on the tube can be predicted as a function of tube length. The geometry is depicted on Figure 1.

Little analytical or computational work on the mass-transfer problem on enhanced surfaces is available in the open literature. Indeed, the author is aware of no multidimensional an-

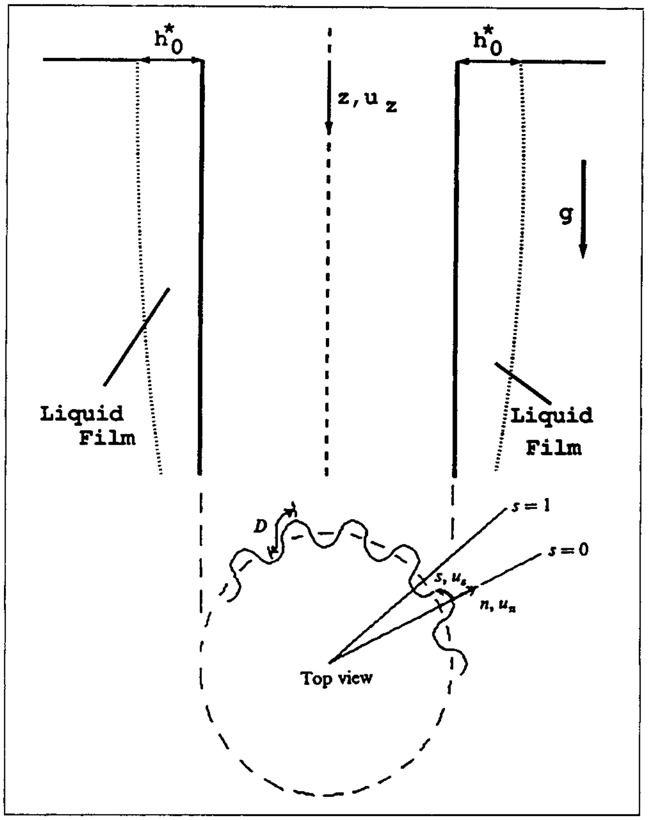

Figure 1. Fluted tube geometry in cross section corresponding to a constant-*s* plane (view from the top).

Because of the very high mass-transfer resistance, the film thickness is very close to that for the case of no mass transfer. Dimensionless coordinates and velocities are shown. From Johnson and Conlisk (1987).

alytical or computational work on the absorption problem on geometrically more complex surfaces than the smooth tube, although Miller and Perez-Blanco (1994) report experimental results on several types of enhanced surfaces. This is perhaps due to the fact that the purely numerical techniques described in the previous work on vertical smooth surfaces (and even eigenfunction expansion techniques) would be very difficult to implement on more complex surfaces and are thus unsuited to the development of an efficient design procedure for any of the enhanced tube geometries currently being considered.

As mentioned above, it has been well known for a number of years that an axially fluted surface enhances the heat-transfer rate in a falling liquid film significantly when compared with the results for a smooth cylindrical surface (Nusselt, 1916; Gregorig, 1954). A significant amount of work is available on the fluid and heat-transfer problems for a pure fluid (Johnson and Conlisk, 1987; Joos, 1984; Honda and Fuji, 1984; Mori et al., 1979; Panchal and Bell, 1979; Sideman and Levin, 1979; Webb, 1979), and the characteristics of this work are discussed extensively by Johnson and Conlisk (1987). Until the work of Joos (1984) and Johnson and Conlisk (1987), however, the precise physical mechanism for the enhancement of heat transfer was unclear. In particular, Johnson and Conlisk (1987) show that, in the case of condensation of a pure fluid in the laminar flow regime, the fluid is driven by surface tension into the troughs (Figure 1), and the film asymptotically reaches a uniform thickness over the majority of the film surface except for a small region near the troughs which steadily thickens because of the continuous inflow. In this work, we consider the same geometry and inquire whether the same is true for the case where a vapor undergoes a phase change at the boundary of a liquid *mixture* film. Two cases are considered here. In the first case, a pure vapor is absorbed into a liquid mixture having one of the components corresponding to that of the vapor. A typical mixture in this class is a lithium-bromide-water mixture, which is presently being used as the working fluid in experimental gas-fired heat pump systems. This process will be termed absorption. In the second, the vapor is also a mixture and the two components in the vapor coincide with the two components in the liquid. A typical mixture in this class is an ammonia-water mixture. This will be termed condensation.

The rate of absorption or condensation of vapor into the film depends strongly on the local flow field. For the vertical smooth tube, at the flow rates of interest, the flow is of the lubrication type and analytical closed-form solutions for the velocity field may be easily obtained outside of the entrance region of the tube (Conlisk, 1991, 1992). Conlisk (1992) has developed a design procedure for the absorption of water into a lithium-bromide-water mixture on a smooth tube which requires very little numerical work and compares well with the experimental data of Miller (1991, 1992) at Oak Ridge National Laboratories (ORNL). In the parameter range of interest, he has shown that mass transfer takes place within a thin layer near the liquid-vapor interface while heat transfer occurs throughout the entire film. In this work, the introduction of new length scales due to the fluting of the surface changes the structure of the heat-transfer problem so that, in a certain parameter range, the major portion of the overall heat transfer may take place near the interface as well. This "thermal boundary layer" is thicker than the "mass-transfer boundary layer"; this particular "boundary layer" structure does not appear to have been recognized in problems in this area. The term "boundary layer" is used in the generalized sense to mean a regime of rapid change in mass fraction and temperature, which, in this case, occurs at the liquid-vapor interface (Figure 1).

The fluting of the surface introduces two additional length scales (in addition to the film thickness h_0^*) into the problem. These are the crest to trough distance D and the length scale L where L is the tube length over which the flow varies as a result of the effect of surface tension. Indeed, the basic physical effect of the flutes is to induce the flow to vary on a length scale much shorter than the length of the tube. Physical arguments suggest that an enhancement effect which occurs in the heat-transfer process may also occur for the mass transfer.

The usual balance for mass transfer in many systems means that Fick diffusion is balanced by convection of the bulk fluid (Bird et al., 1960). The driving potential for the classical Fick diffusion is a mass fraction or concentration gradient, and the proportionality constant is the diffusion coefficient; this is normally the main physical mechanism for mass transfer in absorption systems. In this case, for the smooth tube, the convective terms in the mass-transfer equation are proportional to $\epsilon Re Sc$ where Sc is the Schmidt number, Re is the Reynolds number based on film thickness, and ϵ is the ratio of the film thickness to the length of the tube. For liquids $Sc \gg 1$, which implies that any mass transfer is confined to the region immediately adjacent to the free surface of the liquid film.

The mass-transfer rate depends in part on the overall driving potential for Fick's law. The overall driving potential is characterized by the overall difference between the surface mass fraction and the mass fraction in the bulk of the film. The parameter characterizing this potential is (Acrivos, 1962):

$$B = \frac{\omega_{A\mathrm{Sin}} - \omega_{A\mathrm{BULK}}}{1 - \omega_{A\mathrm{BULK}}} \tag{1}$$

and for $B \ll 1$ and $\epsilon ReSc \gg 1$ (the regime of practical interest), for the smooth tube, the film thickness is constant to leading order and very little mass transfer will take place. Note that the definition of B differs somewhat from the definition in Conlisk (1992); the definition of Eq. 1 is better suited for linking the $O(1)$ B regime with the small B regime. There is, however, little numerical difference between this definition and that used by Conlisk (1992). In Eq. 1, $\omega_{A\mathrm{Sin}}$ is the mass fraction of species A at the film surface at the entrance to the tube and $\omega_{A\mathrm{BULK}}$ is the mass fraction of species A in the bulk of the film.

There have been several papers on the absorption process on smooth surfaces. Grossman (1983) considers the absorption of a pure condensable vapor into a liquid binary mixture. Suggesting that at low absorption rates the film thickness remains constant to leading order, he obtains a solution for the species mass fraction in the liquid film both numerically and using an eigenfunction technique. Grossman (1987) generalizes the results to the case where backdiffusion occurs: concentration or mass fraction differences affect the energy equation. Recently, van der Wekken and Wassenaar (1988) have generalized the problem of Grossman (1983) to include more complex boundary conditions at the wall and at the interface; they also neglect the effect of film thickness variations. Uddholm and Setterwall (1988) consider the effect of a wavy film profile so that the film thickness is not constant. They solve the energy and mass-transfer equations numerically using standard finite difference techniques. Their design procedure is also subject to the specification of several experimentally determined parameters. Andberg and Vliet (1983) compute the solution for the absorption of Libr into water at very low flow rates and include an enthalpy transport term in the energy equation; the mass-transport equation is formally equivalent to the equation solved by Grossman (1983); because of the very low flow rate, mass transfer takes place over the entire film. Their approach is entirely numerical and they comment that closed-form solutions to the problem are unlikely to be found. Grigor'eva and Nakoryakov (1977) consider essentially the same problem as Grossman (1983) and Andberg and Vliet (1983) using an eigenfunction expansion technique. In this work, the tube is smooth and the film thickness is taken to be constant based on physical reasoning that the amount of fluid absorbed into the film is likely to be small; mass transfer is assumed to take place over the entire liquid film. In addition, equilibrium is assumed to hold at the interface. One of the consequences of the boundary layer-type approach employed in Conlisk (1992) is that the specification of a constant film thickness emerges directly from the governing equation of mass transfer due to the very high level of liquid-side, mass-transfer resistance.

There have been several articles on the condensation problem associated with binary mixtures (Sparrow and Marschall, 1969; Kotake and Oswatitsch, 1980; Davis et al., 1984; Denny and Jusionis, 1972). Typical of this work is the focus on the flow in the vapor and the fact that a uniform concentration profile is assumed across the liquid film. Also a constant-temperature interface, which is never observed in practice, is assumed. In this work, the high-Reynolds-number vapor-phase analysis is coupled with a low-Reynolds-number analysis of the condensate liquid film.

The present work focuses on the direct contact absorption and condensation problems, especially on the heat- and mass-transfer processes within the liquid film. Because of the complexity of the fluted surface, the solution for the mass absorbed or condensed on the film is not straightforward as in the smooth tube problem (Conlisk, 1992). From the fundamental solutions to the heat- and mass-transfer problems in the conduction- and convection-dominated heat-transfer regimes obtained, a suitable design procedure for these tubes may be developed. The development of such a design procedure and numerical solutions for the intermediate regime where heat conduction balances heat convection will be reported in the future work.

Fluted Surface

The term fluted surface is given to a tube in which the curvature of the surface is a periodic function of the arc length. Since the specification of the surface is detailed in Johnson and Conlisk (1987), only a short summary will be presented here.

The surface is specified by the Cartesian pair $x = x_0(s)$ and $y = y_0(s)$ where s is the arc length along the perimeter of the tube. Both x_0 and y_0 satisfy ordinary differential equations in s which may be integrated to give:

$$x_0 = \int_0^s \sin K(s)\,ds + X_0, \tag{2}$$

and

$$y_0 = \int_0^s \cos K(s)\,ds + Y_0, \tag{3}$$

where X_0 and Y_0 are the initial points of the curve at $s = 0$ ($s = 1$ denotes the trough) and we have taken $dx_0/ds = 0$ and $dy_0/ds = 1$ at $s = 0$ for convenience. Here,

$$K(s) = \int_0^s |\kappa_b|\,ds, \tag{4}$$

where κ_b is the curvature of the surface and is of the form:

$$\kappa_b = \kappa_1 + \kappa_0 \cos \pi s. \tag{5}$$

For a closed curve, periodicity requires:

$$\kappa_1 \frac{P_0}{D} = 2\pi, \tag{6}$$

where P_0 is the dimensional perimeter of the tube and P_0/D must be an integer equal to twice the number of flutes, say

$2N$. The sign convention on the curvature κ_b is such that it is positive for and largest at the crests and smallest in the troughs.

The types of surfaces governed by Eq. 5 are shown in Figure 2 (Johnson and Conlisk, 1987) for $\kappa_0 = 4$ and $N = 6$, 12, 18, and 24 and for $\kappa_0 = 5$ and $N = 40$ and 60. As N increases, the crest to trough distance D decreases for fixed mean radius and flute amplitude. The validity of this work is limited by the requirement that the film be thin ($h^*/D \ll 1$; h^* is the dimensional film thickness). Typically $D < L$ and in pure fluid applications $N < 60$. For absorption problems, it is likely that N can be much smaller ($N < 10$, say).

For future reference, in the fluted surface coordinate system,

$$\vec{u}\cdot\nabla_1 = u_n\frac{\partial}{\partial n} + \frac{u_s}{G}\frac{\partial}{\partial s} + \frac{\epsilon}{\epsilon_1}u_z\frac{\partial}{\partial z} \tag{7}$$

where $G = G(n, s) = 1 - \epsilon_1 n\kappa_b(s)$.

Absorption Problem

Fluid mechanics

We consider the gravity-driven film flow down the fluted surface of Figure 1. The solution for the velocity field is detailed in Johnson and Conlisk (1987); here as in that work we assume that the convective terms in the Navier-Stokes equations may be neglected and consider only the thin film limit where $\epsilon Re \ll 1$ (Conlisk, 1992). The solution for the dimensionless velocity field is given by:

$$u_z = \left(-1 + \frac{\epsilon}{\epsilon_1}We^{-1}\frac{\partial p}{\partial z}\right)\left(\frac{1}{2}n^2 - nh\right), \tag{8}$$

$$u_s = We^{-1}\frac{\partial p}{\partial s}\left(\frac{1}{2}n^2 - nh\right), \tag{9}$$

$$u_n = -\epsilon_1\int_0^n\left(\frac{\partial u_s}{\partial s} + \frac{\epsilon}{\epsilon_1}\frac{\partial u_z}{\partial z}\right)dn. \tag{10}$$

In these equations, $We = \rho g D^2/\sigma$, $\epsilon_1 = h_0^*/D$, and $\epsilon = h_0^*/L$. Here, We is a Weber number, g is the acceleration due to gravity, ρ is the mixture density, and σ is the surface tension. Also h_0^* is the inlet average film thickness. The velocity scale is the characteristic velocity associated with the Nusselt solution:

$$U = \rho g h_0^{*2}/\mu,$$

where μ is the viscosity, and each of the coordinate variables has been scaled on the corresponding length (n on h_0^*, s on D, and z on L). The length L is determined by the influence of surface tension and is the axial length scale; this scale is determined by a balance between the influence of gravity and surface tension and is $L = WeD$ or $\epsilon/\epsilon_1 = We^{-1}$. In general, $L \geq D$, but much less than the length of the tube, which is the main advantage of the fluted tube: the axial length scale over which the film flow may vary is much shorter than for the smooth tube thus inducing higher film gradients, more mixing, and, in principle, more absorption. For the condensation problem (for a pure fluid) on the fluted tube, the film thickness asymptotes to a constant value as distance down the tube increases, rather than increasing as $(distance)^{1/4}$ as in the smooth tube problem. This fact is important for the heat-transfer problem, since in general the heat-transfer rate is proportional to the film thickness.

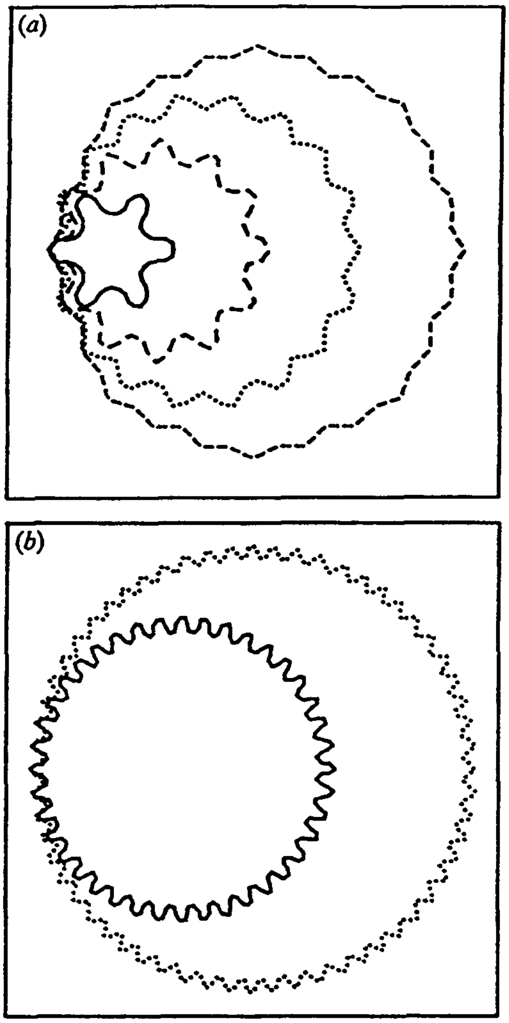

Figure 2. Cross-sectional view of the fluted surface: (a) $\kappa_0 = 4$ and $N = 6, 12, 18, 24$; (b) $\kappa_0 = 5$ and $N = 40, 60$.

From Johnson and Conlisk (1987).

In these thin-film problems, the pressure is, to leading order, proportional to the curvature and from Johnson and Conlisk (1987), $p \approx \kappa_b$. The film thickness is calculated by balancing the mass flux down the tube with that absorbed. Because of the low rate of absorption of water into the film, to leading order, the film thickness may be calculated based on a vanishing mass flux at the interface. In this case, the film thickness satisfies the equation:

$$\frac{\partial h_0^3}{\partial z} - \frac{\partial}{\partial s}\left(h_0^3\frac{\partial \kappa_b}{\partial s}\right) = 0. \tag{11}$$

The solution for h_0 is easily found by standard methods (Zachmanoglou and Thoe, 1976) and the result is:

$$h_0 = \left(\frac{\kappa_b'(s_0)}{\kappa_b'(s)}\right)^{1/3}h_i(s_0) \text{ on } z = -\int_{s_0}^{s}\frac{ds}{\kappa_b'}, \tag{12}$$

for $0<s<1$ and $h_0 = h_i e^{(1/3)\kappa''_{b_0} z}$ for $s=0, 1$. Equation 12 is Eq. 53 of Johnson and Conlisk (1987) in which h is scaled only by the inlet film thickness. Here, h_0 is the leading-order surface-tension-driven component of the film thickness variation ($h = h_0 + \ldots$). On the crest $s=0$, $\kappa''_{b_0} = -\pi^2\kappa_0$ and at the trough $s=1$, $\kappa''_{b_0} = \pi^2\kappa_0$ and consequently the film thins at the crest and thickens at the trough. From Eq. 5,

$$\kappa'_b = -\pi\kappa_0 \sin \pi s. \tag{13}$$

For the type of surface described by Eq. 13, the solution for h_0 is particularly simple:

$$h_0 = \left\{\frac{\sin \pi s_0}{\sin \pi s}\right\}^{1/3} h_i(s_0) \text{ on } z = -\frac{1}{\pi^2\kappa_0} \ln\left\{\frac{\tan \frac{1}{2}\pi s}{\tan \frac{1}{2}\pi s_0}\right\}. \tag{14}$$

The solutions described here indicate that the fluid passes from the crests to the troughs under the action of surface tension. Because the fluid is constantly moving off the crests, the film thickness does not grow as z increases, but asymptotes to a constant value. Typical film profiles for h_0 given by Eq. 14 for $h_i(s_0) = 1$ are shown in Figure 3 for ten flutes and $\kappa_0 = 2$, as well as one crest to trough regime. Note the rapid thinning of the film near the crest with a corresponding rapid thickening in the trough. The validity of this work is limited by the requirement that the film be thin ($h^*/D \ll 1$; h^* is the dimensional film thickness). This requirement is first violated only in a narrow region near the trough, and the remainder of the film remains thin.

Heat transfer

Depending on the properties of the fluted surface (such as number of flutes, curvature, and mean radius), the fluid properties, and the incoming flow rate, heat transfer to the tube wall will span the range from conduction-dominated to convection-dominated. The nature of the temperature depends on the single parameter $\epsilon RePr$ where Re is the Reynolds number defined by $Re = Uh_0^*/\nu$ where ν is the kinematic viscosity and Pr is the Prandtl number. For the case of a thin film, gradients of temperature in the z and s directions are much smaller than the gradients in the direction normal to the fluted surface (n) and in this case the energy equation is given by:

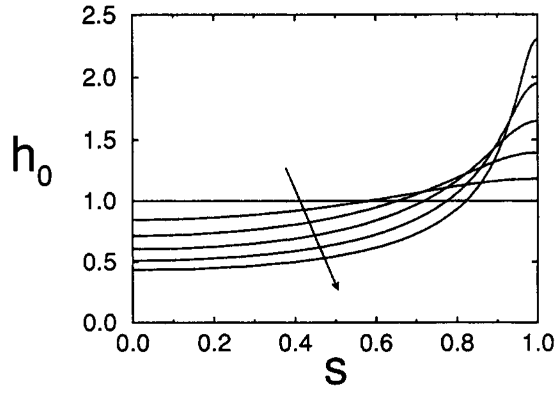

Figure 3. Film profiles for the leading order variation h_0 for $N = 10$ flutes as a function of s for $z = 0$, .5, 1, 1.5, 2, 2.5.

The arrow denotes the direction of increasing z.

$$\frac{\partial^2\theta}{\partial n^2} = \epsilon RePr(\vec{u}\cdot\nabla)\theta, \tag{15}$$

where θ is the dimensionless temperature which will be defined below. Also, from Eq. 7,

$$\vec{u}\cdot\nabla = Weu_s\frac{\partial}{\partial s} + u_z\frac{\partial}{\partial z}.$$

The boundary conditions are given by:

$$\theta = \theta_w \text{ at } n=0; \tag{16}$$

$$\theta = 0 \text{ at } z=0. \tag{17}$$

At the interface, we have, to leading order,

$$Ja\frac{\partial\theta}{\partial n} = -\alpha\epsilon RePr\dot{m}_a \text{ at } n=h, \tag{18}$$

where $Ja = c_p\Delta T/h_{abs}$ where c_p is the specific heat, and $\dot{m}_a$ is the absorbed mass flux, and h_{abs} is the heat of absorption. In what follows we consider each parameter range separately.

$\epsilon RePr \ll 1$

In this case, energy-transfer problem is conduction-dominated to leading order and the solution may be written:

$$\theta(n, s, z) = (\theta_S - \theta_W)\frac{n}{h} + \theta_W \tag{19}$$

where θ_S is the film surface temperature and is unknown, and θ is the scaled temperature defined by:

$$\theta = \frac{T - T_{W\text{in}}}{T_{S\text{in}} - T_{W\text{in}}}. \tag{20}$$

$T_{W\text{in}}$ is the wall temperature at the entrance to the tube and $T_{S\text{in}}$ is the film surface temperature there. Because the heat transfer is conduction-dominated, we cannot satisfy the initial condition at the entrance to the tube. The value of θ_S is determined by satisfying an equilibrium condition at the liquid-vapor interface and this is discussed in detail in the next section.

$\epsilon RePr \sim 1$

In this case, a numerical solution is required; however, the problem is not as straightforward as the case of the vertical smooth tube as discussed by Conlisk (1992). To close the problem, the mass flux $\dot{m}_a$ should be related to the temperature field (Conlisk, 1992). In contrast to the vertical smooth tube, this leads to a complicated boundary condition for which a numerical solution of the type described by Conlisk (1992) is not directly applicable. Consequently, numerical solutions for this parameter range will be reported in the future work. Note that for $\epsilon Re \sim 1$ numerical solutions for the velocity field must be calculated as well.

$\epsilon RePr \gg 1$

In this parameter range, the convective terms in the energy equation dominate except in a thin layer near the interface where convection and diffusion balance. From Eq. 15, the two terms balance in a thin layer of width $O(\delta_1^{1/2})$ where $\delta_1 = 1/\epsilon RePr$; the bulk of the heat transfer thus takes place very near the interface. Consequently, writing $\hat{n} = (1 - n/h)/\delta_1^{1/2}$ the energy equation becomes:

$$\frac{\partial^2 \theta}{\partial \hat{n}^2} = h^2 (\vec{u} \cdot \nabla) \theta, \tag{21}$$

and where now

$$\theta = \frac{T - T_{\text{BULK}}}{T_{\text{Sin}} - T_{\text{BULK}}}. \tag{22}$$

In this equation the subscripts S and BULK refer to interface and bulk, respectively, and T_{BULK} is assumed constant. In addition, the velocity normal to the solid surface has been omitted in the definition of $\vec{u} \cdot \nabla$. This will be shown rigorously in the next section, but this result is due to the very large mass-transfer resistance at the interface.

Because the scaled thermal layer is thin, the boundary condition on the temperature at the interface is formally equivalent to that for the smooth tube and is given by (Conlisk, 1992):

$$Ja \delta_1^{1/2} \left. \frac{\partial \theta}{\partial \hat{n}} \right|_{\hat{n}=0} = h \alpha \dot{m}_a, \tag{23}$$

where $\alpha = \rho w / \rho$ where ρ_w and ρ are the water and mixture densities, respectively; ρ and ρ_w are taken at reference temperatures and α is assumed constant; $\dot{m}_a$ is the absorbed mass flux. The domain in s is from 0 to 1 by definition where $s = 0$ corresponds to the crest of the flutes and $s = 1$ corresponds to the trough. At these points, because of symmetry we must have:

$$\frac{\partial \theta}{\partial s} = 0 \text{ at } s = 0,\ 1. \tag{24}$$

Finally, the initial condition is that $\theta = 0$ at $z = 0$ and $\theta \to 0$ as $\hat{n} \to \infty$. Specification of the boundary conditions at the interface for both the temperature and the mass fraction still does not fix the solution for either quantity.

The solution to Eq. 21 may be obtained as follows. Since the temperature gradient is specified at the interface and $\hat{n}$ ranges from 0 to ∞, a Fourier cosine transform (Churchill 1972) may be employed to reduce Eq. 21 to a first-order inhomogeneous partial differential equation subject to an initial condition at $z = 0$. Define the transform as:

$$\hat{f}(k) = \int_0^\infty f(x) \cos kx dx;$$

then the transform of Eq. 21 is given by:

$$h^2 \left[We u_s(h, s, z) \frac{\partial \hat{\theta}}{\partial s} + u_z(h, s, z) \frac{\partial \hat{\theta}}{\partial z} \right] = -k^2 \hat{\theta} - \frac{h \alpha}{Ja \delta_1^{1/2}} \dot{m}_a, \tag{25}$$

where $\Delta T = T_{\text{Sin}} - T_{\text{BULK}}$. The solution to Eq. 25 is found by standard techniques similar to those employed in the calculation of the film thickness; along the characteristics, Eq. 25 becomes an ordinary differential equation and specifically,

$$\frac{d\hat{\theta}}{dz} = \frac{-k^2 \hat{\theta} - \dfrac{h \alpha \dot{m}_a}{Ja \delta_1^{1/2}}}{h^2 u_z} \text{ on } z = -\int_{s_0}^{s} \frac{ds}{\kappa_b'}. \tag{26}$$

To obtain a solution we note that at the interface $We u_s = -1/2\, h^4 (\partial p / \partial s)$ and $u_z = h^2/2$; using the expression for u_z in Eq. 26 the solution may be represented in transform space by:

$$\hat{\theta} h^4 = \frac{-2\alpha}{Ja \delta_1^{1/2}} e^{-\int_0^z A(\zeta) d\zeta} \int_0^z h \dot{m}_a e^{\int_0^\zeta A d\zeta_1} d\zeta \text{ on } z = -\int_{s_0}^{s} \frac{ds}{\kappa_b'}, \tag{27}$$

where

$$A(\zeta) = 2k^2 h^{-4} - 4h^{-1} \frac{dh}{d\zeta}, \tag{28}$$

valid for $s \neq 0$ or 1. For $s = 0$ or 1, we must return to Eq. 21 and put $\partial \hat{\theta} / \partial s = 0$ there. In this case, the solution has a form which is equivalent to Eq. 27, provided that the proper expression for the film thickness h is used.

It is a simple matter to invert Eq. 27 and the result is:

$$\theta = \frac{-2\alpha}{(\delta_1 \pi)^{1/2} Ja} \int_0^z \frac{\dot{m}_a}{h^3 I} e^{-\hat{n}^{2/8I}} d\zeta \text{ on } z = -\int_{s_0}^{s} \frac{ds}{\kappa_b'}, \tag{29}$$

where

$$I^2 = \int_\zeta^z h^{-4} d\zeta_1. \tag{30}$$

It is interesting to note that apart from scale factors and the fact that the film thickness varies to leading order, the solution (Eq. 29) is very similar to the solution for the smooth tube as given by Conlisk (1992).

At this point, pending consideration of the mass-transfer equation and the interfacial conditions, the solution (Eq. 29) will be discussed later.

Mass transfer

The scaled mass-transfer equation is very similar to the scaled energy equation:

$$\frac{\partial^2\Omega}{\partial\bar{n}^2}=h^2(\vec{u}\cdot\nabla)\Omega, \tag{31}$$

where

$$\Omega=\frac{\omega_A-\omega_{A\mathrm{BULK}}}{\omega_{A\mathrm{Sin}}-\omega_{A\mathrm{BULK}}}, \tag{32}$$

and $\bar{n}=(1-n/h)/\delta^{1/2}$ and $\delta=1/\epsilon ReSc$. In Eq. 31, species A refers to, say, water in a libr-water mixture.

In the scaled variable $\bar{n}$, the boundary condition at the interface becomes:

$$\delta^{1/2}B\,\frac{\partial\Omega}{\partial\bar{n}}=\alpha h\dot{m}_a(1-B\Omega)\ \text{at}\ \bar{n}=0. \tag{33}$$

For small changes in composition, B is small; this is the case for libr-water mixtures of interest here. Equation 31 balances for $B\to 0$; however, the boundary condition at $\bar{n}=0$ (Eq. 33) does not. The conclusion which must be reached is that since B and δ are both small, then

$$\dot{m}_a\sim\delta^{1/2}B; \tag{34}$$

that is, the lack of an $O(1)$ mass fraction difference and the fact that δ is small limit the amount of absorption at the interface: the liquid-side mass-transfer resistance is very large. Consequently, the velocity in the direction normal to the wall at the interface (the dimensionless mass flux; see Eq. (7) is very small [of $O(\delta^{1/2}B)$] and may be omitted in the $\vec{u}\cdot\nabla$ term in Eqs. 21 and 31. Because the mass-transfer resistance is so large, some other mechanism must be introduced to compensate for this, and the ability of the fluted surface to induce film gradients is believed to be crucial to the enhancement process. Moreover, while the solution to these types of coupled problems hinges on the solution to the energy equation, (Eq. 33) with the consequence (Eq. 34) indicates that the entire process is mass-transfer-limited. One implication of this fact is that to leading order, the film thickness $h=h(s,z)$ may be replaced by h_0 as given by Eq. 14.

The other boundary conditions associated with Eq. 31 are similar to those for the temperature, in particular, by symmetry

$$\frac{\partial\Omega}{\partial s}=0\ \text{at}\ s=0,\ 1. \tag{35}$$

Finally, the initial condition is that $\Omega=0$ at $z=0$ and $\Omega\to 0$ as $\bar{n}\to\infty$. The solution to Eq. 31 may be obtained in a manner similar to that for the temperature since $\bar{n}$ ranges from 0 to ∞ as well. Then, the transform of Eq. 31 is given by:

$$h^2\left[We\,u_s(h,s,z)\,\frac{\partial\hat{\Omega}}{\partial s}+u_z(h,s,z)\,\frac{\partial\hat{\Omega}}{\partial z}\right]=-k^2\hat{\Omega}-h\alpha\,\frac{\dot{m}_a}{\delta^{1/2}B}. \tag{36}$$

Note the similarity between Eqs. 25 and 36. As with the temperature, the solution is given by:

$$\hat{\Omega}h^4=-2\alpha e^{-\int_0^z A(\zeta)d\zeta}\int_0^z h\dot{m}_{a0}e^{\int_0^{\zeta}A\,d\zeta_1}\,d\zeta\ \text{on}\ z=-\int_{s_0}^{s}\frac{ds}{\kappa_b'}, \tag{37}$$

where

$$A(\zeta)=2k^2h^{-4}-4h^{-1}\,\frac{dh}{d\zeta}, \tag{38}$$

and $\dot{m}_a=\delta^{1/2}B\dot{m}_{a0}+\ldots$, for $s\neq 0$ or 1; similar comments apply to the form of the solution on $s=0$, 1 as noted in connection with the temperature solution.

The inversion of Eq. 37 results in:

$$\Omega=-\frac{2\alpha}{\pi^{1/2}}\int_0^z\frac{\dot{m}_{a0}}{h^3I}\,e^{-\bar{n}^2}/8I\,d\zeta,\ \text{on}\ z=-\int_{s_0}^{s}\frac{ds}{\kappa_b'}, \tag{39}$$

The mass of water absorbed may be calculated directly from the interface conditions on the temperature, the mass fraction, and the equilibrium condition. These issues are discussed in detail next. Before discussing the closure of the solution, it is useful to describe the boundary layer structure which is obtained near the interface based on the case where $\delta\ll\delta_1\ll 1$. The previous work suggests that there is a set of mass-transfer and thermal boundary layers oriented as shown in Figure 4. Since $\delta/\delta_1=Pr/Sc=Le\ll 1$ where Le is the Lewis number, the mass-transfer layer is much thinner than the thermal layer. Note from Eqs. 29 and 39 that the solutions for the mass fraction and the temperature in these layers are of similar form and are coupled through the interface conditions to be described next.

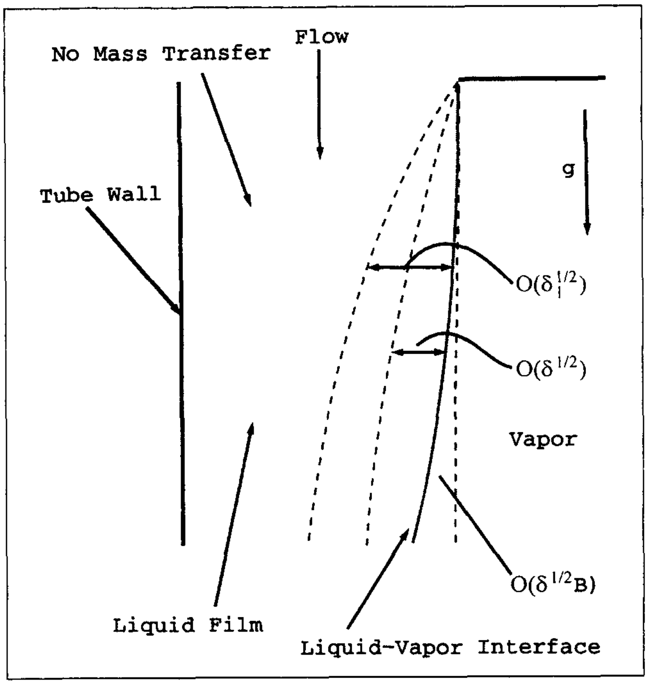

Figure 4. Boundary layer structure in cross section corresponding to a constant-s plane at the liquid-vapor interface for the fluted tube.

The mass-transfer and thermal layers on the liquid-vapor interface are greatly expanded for clarity.

Interface conditions

Specification of the boundary conditions at the interface for both the temperature and the mass fraction still does not fix the solution for either quantity. To do this, equilibrium will be assumed at the interface, and the temperature and mass fraction are assumed to be related linearly as in Grossman (1983); this is a good approximation for libr-water mixtures for the limited range of mass fraction considered in practical applications (Conlisk, 1992), and the equilibrium condition takes the form:

$$\omega_{AS}=C_1T_s+C_2, \tag{40}$$

where C_1 and C_2 are constants obtained by curve-fitting experimental data in the mass fraction range of interest. We will examine here the cases where $\epsilon RePr \ll 1$ and $\epsilon RePr \gg 1$, where a fully numerical solution is required for $\epsilon RePr \sim 1$. The equilibrium equation for the scaled mass fraction and temperature is the same as for the smooth tube:

$$\Omega_S=\frac{C_1\Delta T}{\Delta\omega}\,\theta_S+\beta, \tag{41}$$

where

$$\Delta T=T_{S\text{in}}-T_{W\text{in}},\ \text{for } \epsilon RePr\ll 1, \tag{42}$$

and

$$\Delta T=T_{S\text{in}}-T_{\text{BULK}},\ \text{for } \epsilon RePr\gg 1, \tag{43}$$

and

$$\Delta\omega=\omega_{S\text{in}}-\omega_{A\text{BULK}}. \tag{44}$$

The quantity β is defined by:

$$\beta=(C_1T_{W\text{in}}+C_2-\omega_{A\text{BULK}})/\Delta\omega,\ \text{for } \epsilon RePr\ll 1, \tag{45}$$

and

$$\beta=(C_1T_{\text{BULK}}+C_2-\omega_{A\text{BULK}})/\Delta\omega,\ \text{for } \epsilon RePr\gg 1. \tag{46}$$

For $\epsilon RePr \sim 1$, the definitions of the quantities for $\epsilon RePr \ll 1$ may be used.

$\epsilon RePr \ll 1$

Using the expression for the temperature in Eq. 19 with $h=h_0+\ldots$, and substituting for the film surface temperature using the boundary condition (Eq. 18), and the solution for the mass fraction from Eq. 39, the equilibrium condition becomes:

$$-\frac{2\alpha}{\pi^{1/2}}\int_0^z \frac{\dot m_{a0}}{h_0^3I_0}\,d\zeta=\frac{1}{C}\,(\theta_W-Ah_0\dot m_{a0})+\beta,\ \text{on } z=-\int_{s_0}^{s}\frac{ds}{\kappa_b'}, \tag{47}$$

where we have replaced h by the leading order variation h_0 to define I_0 in Eq. 47. Also,

$$C=\frac{\Delta\omega}{C_1\Delta T}, \tag{48}$$

and

$$A=\frac{\delta^{1/2}B\epsilon RePr\alpha}{Ja}. \tag{49}$$

Equation 47 is a linear integral equation which may be solved for $\dot m_{a0}$; however, the solution is not straightforward because of the appearance of the film thickness term h_0^3I under the integral sign which obviates the use of the Laplace transform for the solution. Nevertheless, a solution may be calculated numerically.

The leading order film thickness may be calculated based on a vanishing absorption mass flux; this is confirmed by the fact that the leading order mass flux is small and

$$\dot m_a=\delta^{1/2}B\dot m_{a0}+\ldots, \tag{50}$$

to leading order. Mathematically, this means that the film thickness changes due directly to the absorption process are small; anticipating this, we expand the film thickness as:

$$h=h_0+\delta^{1/2}Bh_1+\ldots, \tag{51}$$

where h_0 is given by Eq. 14. Then, calculating the quantity $(\vec u\cdot\nabla_1)h$, the result is, to second order,

$$\frac{\partial h_1}{\partial z}-\frac{\partial}{\partial s}\left(\frac{\partial p}{\partial s}\,h_1\right)=-\frac{\dot m_{a0}}{3}-\frac{\partial h_0^2}{\partial z}+\frac{\partial h_0^2}{\partial s}, \tag{52}$$

and this equation gives the relationship between $\dot m_{a0}$ and h_1 since h_0 is known. Equation 52 is easily solved and the result is:

$$h_1(s,z)=e^{\kappa_b''z}\int_0^z e^{-\kappa_b''\zeta}F(s,\zeta)d\zeta,\ \text{on } z=-\int_{s_0}^{s}\frac{ds}{\kappa_b'}, \tag{53}$$

where $F(s, z)$ is the righthand side of Eq. 52.

$\epsilon RePr \gg 1$

In this parameter range, the similarity of the temperature and mass fraction solutions leads to a relatively straightforward solution for the mass flux. Substituting the expansion for the leading order mass flux, $\dot m_a=\delta^{1/2}B\dot m_{a0}+\ldots$ into Eq. 29 and evaluating at $\hat n=0$ and evaluating Eq. 39 at $\bar n=0$ it is readily determined that

$$\theta_s=\frac{BLe^{1/2}}{Ja}\,\Omega_s\ \text{on } z=-\int_{s_0}^{s}\frac{ds}{\kappa_b'}. \tag{54}$$

Using the equilibrium equation above to eliminate the film surface temperature θ_s, the surface mass fraction is given by:

$$\Omega_s = \frac{\beta}{1 - \dfrac{C_1 \Delta T B L e^{1/2}}{Ja \Delta\omega}}, \text{ on } z = -\int_{s_0}^{s} \frac{ds}{\kappa_b'}, \tag{55}$$

and thus, the surface mass fraction depends crucially on the bulk temperature; if the bulk temperature is constant as has been assumed here, so is the surface mass fraction. Note also that for absorption, $\Omega_s > 0$ and this condition places limits on the operating bulk temperature and mass fraction, given the physical constants C_1 and C_2.

Now, the mass fraction at the interface is known from Eq. 55 and evaluating Eq. 39 at $\bar{n} = 0$:

$$\Omega_s = -\frac{2\alpha}{\pi^{1/2}} \int_0^z \frac{\dot{m}_{a0}}{h_0^3 I_0} d\zeta, \text{ on } z = -\int_{s_0}^{s} \frac{ds}{\kappa_b'}. \tag{56}$$

Equation 56 is a linear integral equation for $\dot{m}_{a0}$ and it may be solved as follows. Since I_0 is essentially a function of s and $z - \zeta$, Eq. 56 is a convolution integral with respect to a Laplace Transform in z. Taking the Laplace Transform of Eq. 56:

$$\hat{\Omega}_s = -\frac{2\alpha}{\pi^{1/2}} J(p) M(p), \tag{57}$$

where p is the Laplace Transform variable and

$$J(p) = \int_0^{\infty} \frac{e^{-pt}}{I_0} dt, \tag{58}$$

and where $M(p)$ is the Laplace Transform of $\dot{m}_{a0}/h_0^3$. Equation 57 may be solved for $M(p)$ and inverted, again by convolution and the result is:

$$\dot{m}_{a0} = -h_0^3 \frac{\pi^{1/2}}{2\alpha} \int_0^z \Omega_s(z-t) j(t) dt \text{ on } z = -\int_{s_0}^{s} \frac{ds}{\kappa_b'}, \tag{59}$$

where $j(t)$ is the inverse transform of $1/J$. For T_{BULK} constant, Ω_s is constant on characteristics and may be pulled out of the integral in Eq. 59; thus, $\dot{m}_{a0}$ is directly proportional to Ω_s and a major objective of an optimal design is a maximum Ω_s. Lastly, given $\dot{m}_{a0}$, the correction to the film thickness h_1 may be obtained directly by Eq. 53. The solution is now complete.

Note that the solution (Eq. 59) requires the evaluation of the Laplace Transform of I_0^{-1}. This exercise is nontrivial because of the fact that I_0 is an integral of h_0 and must be evaluated along the characteristics defined in Eq. 14. It appears that the forward transform $J(p)$ must be computed numerically; because of this, the inverse transform must also be calculated numerically. The numerical evaluation of these transforms and the development of a design procedure for these tubes will be presented in subsequent work.

To obtain a feel for the magnitude of some of the parameters, the fluid properties of one of the experimental data sets tested by Miller (1992) for a LiBr-water solution was used to calculate a value for the length L defined by $L = DWe$. The mean radius of the tube was taken to be 0.525 in. (13.335 mm) and the inlet temperature was taken to be 328 K. For an inlet flow rate of 0.72 kg/min, for six flutes and $\kappa_0 = 2$ the length $L = 0.07$ m ($\sim$3 in.). For smaller radii, the length L is considerably decreased. In the Miller experiments on the smooth tube the flow and heat and mass transfer vary on the scale of the length of the tube which is 5 ft (1.5 m). Consequently, the fluted tube induces a variation of the flow on a scale much shorter than in the smooth tube.

Having obtained the solution for the mass flux at the interface, we now extend the results to the case where the vapor phase contains two volatile components.

Condensation Problem

In this section, we extend the methods described previously for the case of one volatile component in the vapor to the case where both components may condense into the liquid film. An example of this type of mixture which is commonly employed in heat pump systems is an ammonia-water mixture.

The vapor condensation rate is controlled by the mass-transfer process in the liquid film and as with the lithium-bromide mixture, at typical operating pressure and temperature, a linear relationship between the film surface mass fraction and film surface temperature obtains within a range of mass fraction typical in applications, and we assume Eq. 40 is still valid. Because both components of the mixture may condense, additional analysis is required to determine the boundary condition at the interface. We begin with the assumption that the total flux of mixture from the vapor is equal to the influx to the liquid:

$$\dot{m}_{cL}^* = \dot{m}_{cV}^* = \dot{m}_c^* \tag{60}$$

where $\dot{m}_c^*$ has units of, say kg/m^2·s. Now consider the expression for the mass flux in a direction normal to the interface, which, for a thin film is in the n direction ($n = n^*/h_0^*$) and is given by (Bird et al., 1960):

$$-\rho D_{ABL} \frac{\partial \omega_{AL}}{\partial n^*} = \dot{m}_{AL}^* - \omega_{AL} \dot{m}_c^*, \text{ at the interface,} \tag{61}$$

where $\dot{m}_{AL}^*$ is the flux of species A at the interface on the liquid side. An equation of the same form obtains on the vapor side. To determine the proper boundary condition, we assume in each of the two flux boundary conditions that the flux of species A is the same from each side ($\dot{m}_{AL}^* = \dot{m}_{AV}^*$) and substituting the vapor side expression into the liquid side expression, we obtain in nondimensional form:

$$-\delta \frac{\partial \omega_{AL}}{\partial n} = -P \frac{\partial \omega_{AV}}{\partial n_1} + (\omega_{AV} - \omega_{AL}) \dot{m}_c \text{ at the interface,} \tag{62}$$

where P is a constant defined by:

$$P = -\frac{\epsilon_2 \mu_V}{\epsilon \mu Re Sc_V}, \tag{63}$$

where $\epsilon_2 = h_0^*/d$, Sc_V is the Schmidt number of the vapor, μ_V is the vapor viscosity, $n_1 = n^*/d$ where d is the global length scale associated with the vapor side. Re is the liquid-side Reynolds number and μ is the liquid viscosity. If Eq. 62 is nondimensionalized, the vapor mass fraction gradient is multiplied by the parameter PB_V where B_V is a mass-transfer driving parameter similar to the definition of B. If the vapor Reynolds number is relatively low [say $Re_V = O(1)$] so that there is no vapor boundary layer, then from Eq. 62 and the scalings of the liquid-side variables in the previous section, the liquid and vapor-side mass-transfer problems are strongly coupled if $PB_V \sim \delta^{1/2}B$ and decoupled if $PB_V \ll \delta^{1/2}B$ and ω_{AV} varies on the scale d. At higher values of the vapor Reynolds number, since the Schmidt number of the vapor is around 1, the mass fraction "boundary layer" on the vapor side is of the same thickness as the fluid boundary layer; in this case, the liquid and vapor mass-transfer problems are strongly coupled if $PB_V Re_V^{1/2} \sim \delta^{1/2}B$ and decoupled if $PB_V Re_V^{1/2} \ll \delta^{1/2}B$. At very high vapor flow rates, $PB_V Re_V^{1/2} \gg \delta^{1/2}B$ and the vapor side controls the entire mass-transfer problem. Generally, L is of the order of 1 in. (25.4 mm), and if the vapor cavity is relatively large so that d can be taken to be the length of the tube [say 1 ft (0.3 m)], then at low vapor Reynolds number P will satisfy, $PB_V \ll \delta^{1/2}B$. Consider the case where $PB_V \ll \delta^{1/2}B$ and neglect the first term of Eq. 62. Note that neglect of this term does not imply that no mass transfer is taking place in the vapor, only that what diffusion mass transfer does take place is small. Note, however, that the mass transfer in the liquid depends crucially on the interfacial vapor mass fraction, which in this case can be obtained from equilibrium considerations.

At this point, we scale the mass fractions ω_{AL} and ω_{AV} to derive the equation corresponding to Eq. 33:

$$\delta^{1/2}B\,\frac{\partial\Omega}{\partial\overline{n}} = \dot{m}_c(1 + B_V\Omega_V - B\Omega),\ \text{at}\ \overline{n} = 0, \qquad (64)$$

where

$$B_V = \frac{\omega_{AVSin} - \omega_{ABULK}}{\omega_{AVBULK} - \omega_{ABULK}} \qquad (65)$$

and

$$\Omega_V = \frac{\omega_{AV} - \omega_{AVBULK}}{\omega_{AVSin} - \omega_{AVBULK}}. \qquad (66)$$

Ω is the scaled mass fraction in the liquid. As before, $\dot{m}_c = \delta^{1/2}B\dot{m}_{c0} + \ldots$ and for small change in vapor mass fraction ($B_V \ll 1$) the boundary condition (Eq. 64) is the same as Eq. 33, and the solutions given previously for small B are valid provided we set $\alpha = 1$. Often, however, larger changes in the vapor mass fraction may occur, and it can happen that $B_V = O(1)$ even when B is small since $Sc_V = O(1)$. In this case, we can write the boundary condition (Eq. 64) to leading order in B as:

$$\delta^{1/2}B\,\frac{\partial\Omega}{\partial\overline{n}} = \dot{m}_c(1 + \beta_V),\ \text{at}\ \overline{n} = 0. \qquad (67)$$

where $\beta_V = B_V\Omega_V$. Thus, the solution for Ω is given by Eq. 39 with $\dot{m}_{a0}$ replaced with $\dot{m}_{c0}(1 + \beta_V)$. The temperature distribution remains the same as in the one-volatile-component problem. For the case of $\epsilon RePr \ll 1$, it is a simple matter to generate the equation corresponding to Eq. 47; for $\epsilon RePr \gg 1$ the equation corresponding to Eq. 56 is (note that β_V does not appear in the solution for the temperature and so Eq. 54 no longer holds):

$$-\frac{C_1\Delta T}{\Delta\omega}\frac{2}{\pi^{1/2}}\frac{Le^{1/2}}{Ja}\int_0^z \frac{\dot{m}_{c0}}{h_0^3 I_0}\,d\zeta + \beta = -\frac{2}{\pi^{1/2}}\int_0^z \frac{\dot{m}_{c0}(1 + \beta_V)}{h_0^3 I_0}\,d\zeta\ \text{on}\ z = -\int_{s_0}^{s}\frac{ds}{\kappa_b'}. \qquad (68)$$

The difference between the condensation case and the absorption case for $B_V O(1)$ is that the function $\beta_V = \beta_V(z)$ is unknown and related to the vapor mass fraction at the interface defined above. Once this quantity is known, the equilibrium relationship between the unscaled liquid and vapor mass fractions may be employed to calculate a value for Ω_V at the interface. The form of this expression means that Eq. 68 is a nonlinear integral equation for $\dot{m}_{c0}$ which is not easy to solve in the general case. Nevertheless, once the solution for $\dot{m}_{c0}$ is obtained, the solution for the perturbation to the leading order film thickness h_1 may be obtained from Eq. 52.

Generally speaking, for an ammonia-water mixture the length scale L is smaller than for the libr-water mixture. At a temperature of 400 K, and a flow rate of 0.027 kg/min, for four flutes and $\kappa_0 = 2$, $L = 0.022$ m [almost 1 in. (25.4 mm)]; for this mixture, heat transfer tends to be more conduction-dominated than the libr-water mixture because of the much lower value of the Prandtl number.

Summary

We have formulated and solved the problem of absorption of a single fluid into a falling film binary mixture on a fluted tube and extended the results to the condensation problem where both components of the vapor may change phase. The type of tube described here is an example of a general class of enhanced tubes in which the mass-transfer rate is expected to be much greater for a given length than for a smooth tube. The entire coupled heat- and mass-transfer problem hinges on the solution of the energy equation, although the entire process is mass-transfer-limited (see boundary condition of Eq. 33). The nature of the heat transfer is determined by the magnitude of the parameter $\epsilon RePr$ which is a composite dimensionless number dependent on flow rate, geometry of the tube, and fluid properties. For $\epsilon RePr \ll 1$, conduction dominates convection and the absorbed mass flux satisfies a linear integral equation of an Abel-type with a nontrivial kernel. For $\epsilon RePr \sim 1$, heat conduction balances convection and the solution for the temperature must be computed numerically. For $\epsilon RePr \gg 1$, heat transfer takes place in a thin layer of width $O(\epsilon RePr)^{-1/2}$ near the liquid-vapor interface. In this case, the absorbed mass flux is given by the integral expression (Eq. 59). For all the mixtures of interest, mass transfer takes place in a thin layer near the liquid-vapor interface since the Schmidt number is very large (the Lewis number $Le \ll 1$). Although it has been stated here that two mixtures, in particular, are of interest in these problems (lithium bromide-water and ammonia-water), this work is clearly applicable to a much wider range of fluid mixtures. In addition, there is nothing in this theory to suggest that the fundamental structure of the heat- and mass-transfer problems does not apply to desorption as well.

The absorption or condensation of vapor has been shown to be controlled by a mass-transfer driving parameter (B) which is small in this context. It is believed that this work is one of very few efforts to solve the absorption and mixture condensation problems on enhanced surfaces. These solutions may be employed to develop a design program for the axially fluted tube as has been done for the vertical smooth tube (Conlisk, 1994), and this program is being developed.

To investigate the enhancement process it is important to note that in general there is a significant increase in surface area for the fluted tube. Thus, there is a natural enhancement factor associated with this passive increase in area. True enhancement can only occur if the ratio of total mass absorbed for the fluted tube to that of the smooth tube is greater than the area ratio. To investigate whether this is true, we note from the results that the dimensionless mass absorbed per unit area $\dot{M}_a = \dot{m}_a \sim \epsilon B\delta^{1/2}$; this indicates that the mass flux increases for the same values of the Schmidt number and B with decreasing values of L (increasing surface tension), provided that the Reynolds number remains approximately constant. It is useful to note that the value of ϵ is much greater than that for the smooth tube, because the surface tension controls the axial length scale; this length scale (L) is usually much less than the total length of the tube. For the example case of the lithium-bromide water mixture discussed earlier, a conservative estimate of the enhancement factor based on the comparison of the quantity $\epsilon\delta^{1/2}B$ with that of a 2-ft-long (0.6-m-long) smooth tube is that mass transfer could be enhanced by a factor of two, although confirmation of this estimate must await the numerical solutions to come.

In this work, solutions for the film thickness have been obtained for the case where there is no vertical pressure gradient within the liquid film. However, the solutions may be extended to the case of an externally imposed constant pressure gradient. Finally it is worthwhile to note that the structure of the mass- and heat-transfer problems does not change at higher flow rates where the convective terms in the momentum equations are important; in this case, numerical solutions are required for the fluid problem, for the calculation of the film thickness, and for the temperature if $\delta_1 = O(1)$. Nevertheless, for δ, $\delta_1 \ll 1$, the solutions for the temperature and for the mass fraction in their respective layers near the liquid-vapor interface are still valid pending computation of the solution for the flow field.

Acknowledgment

The author is grateful to Professor Saleh Tanveer who suggested the use of the cosine transform in the solution for the mass fraction and temperature for $\epsilon RePr \gg 1$.

Literature Cited

Acrivos, A., "The Asymptotic Form of the Laminar Boundary Layer Mass Transfer Rate For Large Interfacial Velocities," *J. Fluid Mech.*, **12**, 337 (1962).

Andberg, J. W., and G. C. Vliet, "Nonisothermal Absorption of Gases into Falling Liquid Films," ASME-JSME Joint Thermal Engineering Conf., **2**, p. 423 (1983).

Bird, R. B., W. E. Stewart, and E. N. Lightfoot, *Transport Phenomena*, Wiley, New York (1960).

Churchill, R. V., *Operational Mathematics*, 3rd ed., McGraw-Hill, New York (1972).

Conlisk, A. T., "Falling Film Absorption on a Spined Cylindrical Tube," Final Report for Subcontract V5875(4772)-1609, Battelle Columbus Labs (1991).

Conlisk, A. T., "Falling Film Absorption on a Cylindrical Tube," *AIChE J.*, **38**(11), 1716 (1992).

Conlisk, A. T., "The Use of Boundary Layer Techniques in Absorber Design," Int. Absorption Heat Pump Conf., New Orleans (1994).

Davis, J. F., H. Tung, and R. S. H. Mah, "Fractionation with Condensation and Evaporation in Wetted Wall Columns," *AIChE J.*, **30**(2), 328 (1984).

Denny, V. E., and V. J. Jusionis, "Effects of Forced Flow and Variable Properties on Binary Film Condensation," *Int. J. Heat Mass Transfer*, **15**, 2143 (1972).

Gregorig, R., "Hautkondensation an Funegewellton Oberflachen bei Berucksichtigung der Oberflachenspannungen," *Z. Angew. Math. Phys.*, **5**, 36 (1954).

Grigor'eva, N. I., and V. E. Nakoryakov, "Exact Solution of Combined Heat and Mass Transfer Problem During Film Absorption," *J. Eng. Phys.*, **33**(5), 1349 (1977).

Grossman, G., "Simultaneous Heat and Mass Transfer in Film Absorption Under Laminar Flow," *Int. J. Heat Mass Transfer*, **26**(3), 357 (1983).

Grossman, G., "Analysis of Interdiffusion in Film Absorption," *Int. J. Heat Mass Transfer*, **30**(1), 205 (1987).

Honda, H., and T. Fuji, "Semi-empirical Equation for Condensation Heat Transfer on Vertical Fluted Tubes," *Fundamentals of Phase Change: Boiling and Condensation*, C. T. Avedisian and T. M. Rudy, eds., ASME Meeting, New Orleans (Dec. 9-14, 1984).

Johnson, R. E., and A. T. Conlisk, "Laminar Film Condensation/Evaporation on a Vertically Fluted Tube," *J. Fluid Mech.*, **184**, 245 (1987).

Joos, F. M., "Thin Liquid Films on Arbitrary Surfaces with Condensation or Evaporation," *Fundamentals of Phase Change: Boiling and Condensation*, C. T. Avedisian and T. M. Rudy, eds., ASME Meeting, New Orleans (Dec. 9–14, 1984).

Kotake, S., and K. Oswatitsch, "Parameters of Binary Mixture Film Condensation," *Int. J. Heat Mass Transfer*, **23**, 1405 (1980).

Miller, W. A., and H. Perez-Blanco, "Vertical-Tube Aqueous LiBr Falling Film Absorption Using Advanced Surfaces," Int. Absorption Heat Pump Conf., New Orleans (1994).

Miller, W. A., private communications (1991, 1992).

Mori, Y., K. Hijikata, S. Huasawa, and W. Nakayama, "Optimized Performance of Condensors with Outside Condensing Surface," *Condensation Heat Transfer*, P. J. Marto and P. G. Kroeger, eds., Heat Transfer Conf., San Diego (Aug. 6-8, 1979).

Nusselt, W., "Die Oberflachenkondensation des Wasserdampfes," *Z. Vereines deutscher Ing.*, **60**, 569 (1916).

Panchal, C. B., and K. J. Bell, "Analysis of Nusselt-Type Condensation on a Vertical Fluted Tube," *Condensation Heat Transfer*, P. J. Marto and P. G. Kroeger, eds., Heat Transfer Conf., San Diego (Aug. 6-8, 1979).

Sideman, S., and A. Levin, "Effect of Configuration on Heat Transfer on a Vertically Grooved Surface," *Desalination*, **31**, 7 (1979).

Sparrow, E. M., and E. Marschall, "Binary, Gravity-Flow Film Condensation," *ASME J. Heat Transfer*, **91**, 205 (1969).

Uddholm, H., and F. Setterwall, "Model for Dimensioning a Falling Film Absorber in an Absorption Heat Pump," *Int. J. Ref.*, **11**, 41 (Jan., 1988).

van der Wekken, B. J. C., and R. H. Wassenaar, "Simultaneous Heat and Mass Transfer Accompanying Absorption in Laminar Flow Over a Cooled Wall," *Int. J. Ref.*, **11**, 70 (Mar., 1988).

Webb, R. L., "A Generalized Procedure for the Design and Optimization of Fluted Gregorig Condensing Surfaces," *ASME J. Heat Transfer*, **101**, 101 (1979).

Zachmanoglou, E. C., and D. Thoe, *Introduction to Partial Differential Equations with Applications*, Williams and Wilkins, Baltimore (1976).

Manuscript received Mar. 5, 1993, and revision received Aug. 9, 1993.